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SHORT COMMUNICATIONS

PROPAGATION OF TORSIONAL SURFACE WAVES IN VISCOELASTIC MEDIUM

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SUMMARY

It is well known that an elastic homogeneous half-space does not allow torsional surface waves to propagate. The present paper attempts to find out the possibility of propagation of such waves in a viscoelastic half-space. The study reveals that although the homogeneous elastic half-space does not allow torsional surface waves to propagate, a viscoelastic half-space does so. The wave is damped due to the viscoelastic parameter. It has also been found that as the viscoelastic parameter decreases, the medium becomes elastic and the torsional surface waves ceases to propagate.

KEY WORDS: torsional surface waves; geophysical prospecting; earthquake damage

INTRODUCTION

The study of surface waves in a half-space is important to seismologists due to its possible applications in geophysical prospecting and in understanding the cause and estimation of damage due to earthquakes. Quite a good amount of information about the propagation of seismic waves is available in Ewing *et al.*[1] A large number of papers have been published in different journals after the publications of this book. In fact, the study of surface waves for homogeneous, non-homogeneous and layered media has been of central interest to theoretical seismologists up to recent times. Of these, the work of Vrettos[2,3] on the study of surface waves in inhomogeneous media is worth mentioning. His study in the above papers provides information on the effect of non-homogenity on the propagation of surface waves due to line loads. Although much information is available on the propagation of surface waves, such as Rayleigh waves, Love waves and Stonely waves, etc., the torsional wave has not drawn much attention and a limited amount of literature is available on the propagation of this wave. Lord Rayleigh in his remarkable paper[4] showed that the isotropic homogeneous elastic half-space does not allow torsional surface waves to propagate. Later on, Meissner[5] pointed out that in an inhomogeneous elastic half-space with quadratic variation of shear modulus and density varying linearly with depth, torsional surface waves do exist. Recently Vardoulakis[6] has shown that torsional surface waves also propagate in Gibson half-space, that is, a half-space whose shear modulus varies linearly with depth and with constant density. Torsional waves in an initially stressed cylinder have been

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studied by Dey and Dutta[7] and the existence and propagation of torsional surface waves in an elastic half-space with void pores has been discussed by Dey *et al.*[8]

So far, no literature is available on the existence of torsional surface waves in a viscoelastic half-space. The present paper attempts this study and concludes that a viscoelastic soil allows the propagation of torsional surface waves. The presence of the imaginary part of the velocity equation suggests that the propagation is damped. The real and damped velocities of the torsional surface wave in a viscoelastic layer have been obtained and the numerical calculations suggest that as the viscoelatic parameter decreases, the overall velocity of the torsional surface wave diminishes.

FORMULATION

To study torsional waves a cylindrical co-ordinate system is introduced, with the z-axis towards the interior of the viscoelastic half-space. The origin of the co-ordinate system is located on the surface of the half-space at the centre of a circular region. A twist of mangitude P has been applied along a circumference of radius a. Let r and θ be the radial and circumferential co-ordinates, respectively. It is assumed that the torsional wave travels in the radial direction and that all mechanical properties associated with it are independent of θ. For torsional waves, $u = w = 0$ and $v = v(r, z, t)$ and the equation of motion for viscoelastic voigt type may be written as[1]

$$\left(\mu + \mu' \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 v}{\partial t^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2}\right) = \rho \frac{\partial^2 v}{\partial t^2} \tag{1}$$

where μ is the modulus of rigidity of the medium and μ' is the viscoelastic parameter.

For the wave propagating along the r direction one may assume the solution of (1) as

$$v = V(z)J_0(Kr)e^{iwt} \tag{2}$$

where V is the solution of

$$V'' - K^2\left[1 - \frac{C_1^2}{C_2^2(1 + iA)}\right]V = 0 \tag{3}$$

in which $C_1 = w/K$, $C_2 = (\mu/\rho)^{1/2}$, $A = w\mu'/\mu$, $w = 2\pi/T$ and J_0 is the Bessel's function of the first kind and of zero order.

The solution of equation (3) satisfying the condition $\mathrm{Lt}_{z\to\infty} V(z) = 0$ is

$$V = [B_1 \cos(\alpha_1\alpha_3 z) - B_2 \sin(\alpha_1\alpha_3 z)]\exp(-\alpha_1\alpha_2 z) \tag{4}$$

where

$$\alpha_1 = \left[\frac{K^2\gamma_1}{C_2^2(1 + A^2)}\right]^{1/2}$$

$$\alpha_2 = \cos\frac{\theta}{2}$$

$$\alpha_3 = \sin\frac{\theta}{2}$$

$$\gamma_1 = [\{C_2^2(1 + A^2) - C_1^2\}^2 + (AC_1^2)^2]^{1/2} \tag{5}$$

$$\theta = \tan^{-1}\left[\frac{A}{(C_2^2/C_1^2 + AC_2/C_1 - 1)}\right]$$

Therefore the final solution of equation (1) may be written as

$$v = [B_1 \cos(\alpha_1\alpha_3 z) - B_2 \sin(\alpha_1\alpha_3 z)]J_0(Kr)\exp(iwt - \alpha_1\alpha_2 z) \tag{6}$$

Boundary conditions

For the viscoelastic medium, the boundary conditions are

$$\left(\mu + \mu'\frac{\partial}{\partial t}\right)\left(\frac{\partial v}{\partial \gamma} - \frac{v}{\gamma}\right) = 0 \quad \text{at } z = 0$$

and

$$\left(\mu + \mu'\frac{\partial}{\partial t}\right)\left(\frac{\partial v}{\partial z}\right) = \frac{P}{2\pi a} \quad \text{at } z = 0, \quad r = a \tag{7}$$

which gives

$$\frac{B_2}{B_1} = -\cot\frac{\theta}{2} \tag{8}$$

and

$$B_1 = \frac{P}{2\pi a(\mu + iw\mu')\,[K^2a/4 + 3K^4a^3/64 - 1/a]} \tag{9}$$

Using equation (9) in equation (8) one gets

$$B_2 = \frac{-P\cot(\theta/2)}{2\pi a(\mu + iw\mu')[K^2a/4 + 3K^4a^3/64 - 1/a]} \tag{10}$$

Substituting the value of B_1 and B_2 in equation (6) one gets

$$v = \frac{P[Y_1\cos(Y_1Kz) + Y_2\sin(Y_1Kz)]}{2\pi a Y_1(\mu + iw\mu')\,[K^2a/4 + 3K^4a^3/64 - 1/a]}J_0(Kr)\exp(-Y_2Kz) \tag{11}$$

where

$$Y_1 = \frac{\alpha_1\alpha_3}{K}$$

and

$$Y_2 = \frac{\alpha_1\alpha_2}{K} \tag{12}$$

The secular equation (11) giving the velocity of the torsional wave in viscoelastic medium is

$$Y_1 = 0$$

which gives

$$\frac{C_1}{C_2} = \left[\frac{(1 + A^2)^{1/2} + 1}{2}\right]^{1/2} \pm i\left[\frac{(1 + A^2)^{1/2} - 1}{2}\right]^{1/2} \tag{13}$$

Equation (13) shows that the torsional wave in a viscoelastic medium will propagate and the velocity will be damped by the presence of the viscoelastic parameter μ'.

In case the medium is elastic $\mu' = 0$ and hence $A = 0$ giving $C_1 = C_2$, meaning that the torsional mode coincides with the shear wave mode in the homogeneous elastic half-space agreeing with the observation of Rayleigh.[4]

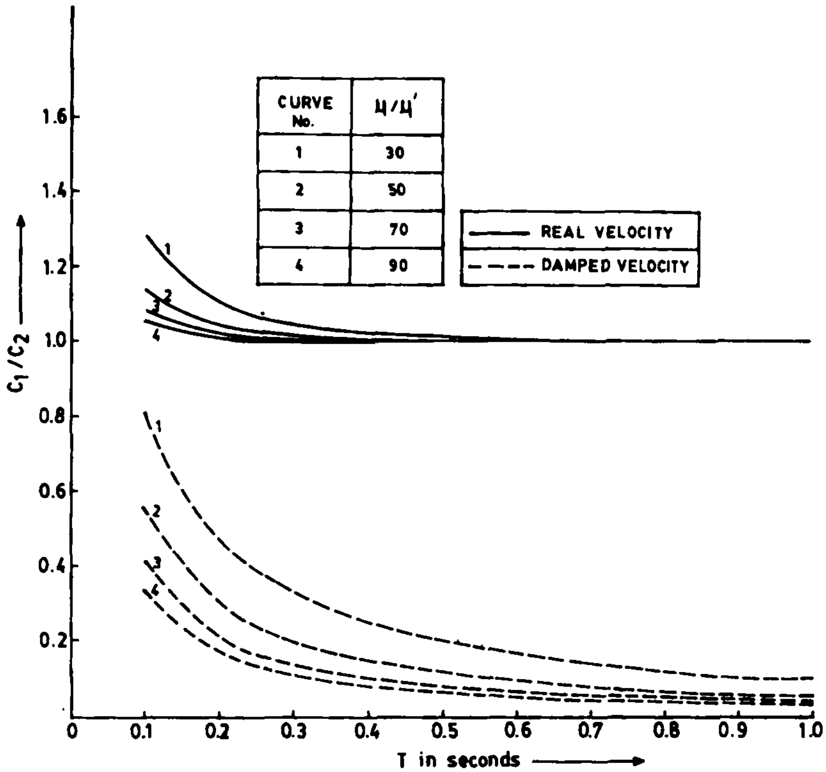

Figure 1. Phase velocity of torsional surface waves in viscoelastic half-space

In a viscoelastic medium the wave is damped and the absorption coefficient is given by[1]

$$\tau = \frac{2\pi^2 \mu'}{\mu C_1 T^2} \text{ per km.} \tag{14}$$

Numerical calculations

The real and damped part of the velocity from equation (13) and the absorption coefficient from equation (14) were calculated numerically for different values of μ'/μ and the results are presented in Figures 1 and 2.

The curves show that the presence of the viscoelastic parameter affects both the real and damped velocity. As μ'/μ increases, both the real and damped velocity decrease but their difference increases showing that the velocity of the torsional wave in viscoelastic soil is less than that of the shear wave velocity in elastic soil.

The nature of the curves in Figure 2 also suggests that the increase in viscoelasticity diminishes the velocity of the torsional surface wave.

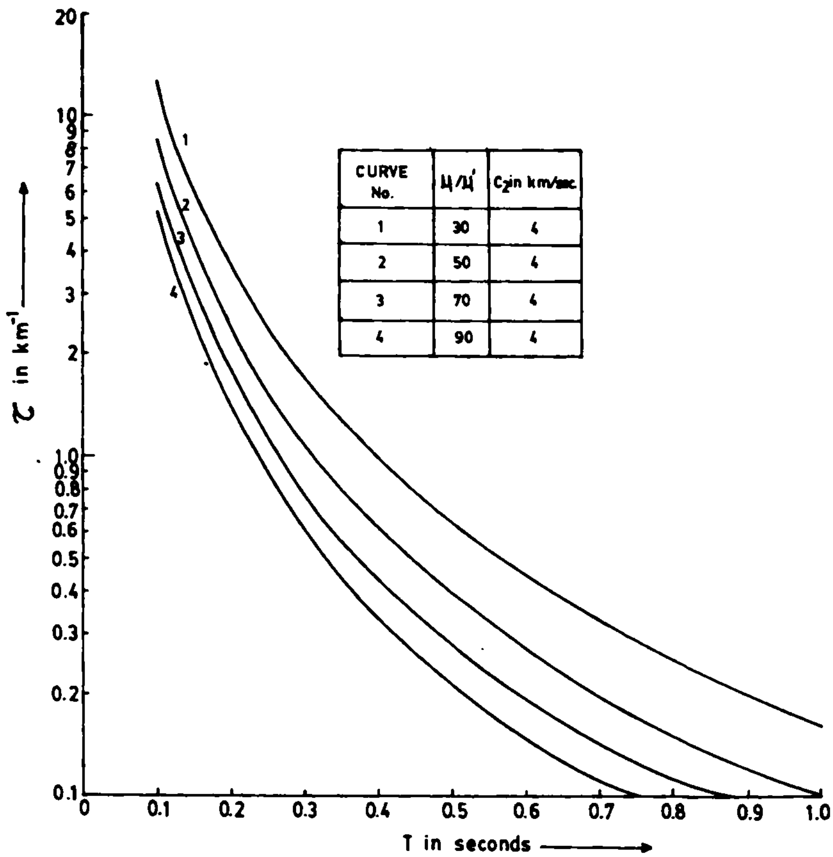

Figure 2. Absorption coefficient of torsional surface waves in a viscoelastic half-space

REFERENCES

1. W. M. Ewing, W. S. Jardetzky and F. Press, *Elastic Waves in Layered Media*, McGraw-Hill, New York, 1957.
2. Ch. Vrettos, 'In-plane vibrations of soil deposits with variable shear modulus: II Line load', *Int. j. numer. anal. methods geomech.*, **14**, 649–662 (1990).
3. Ch. Vrettos, 'In pulse vibrations of soil deposits with variable shear modulus I. Surface waves', *Int. j. numer. anal. methods geomech.*, **14**, 209–222 (1990).
4. Lord Rayleigh, *Theory of Sound*, Dover, New York, 1945.
5. E. Meissner, Elastic oberflachenwellen mit Dispersion in einem inhomogenen Medium, Viertelgahrsschriftder Naturforschenden Gesellschaft in Zurich, Vol. **66**, 1921, pp. 181–195.
6. I. Vardoulakis, 'Torsional surface waves in inhomogeneous elastic media' *Int. j. numer. anal. methods Geomech.*, **8**, 287–296 (1984).
7. S. Dey and D. Dutta, 'Torsional wave propagation in an initially stressed cylinder, *Proc. Indian Nat. Sci. Acad. A.*, **58**, 425–429 (1992).
8. S. Dey, S. Gupta and A. K. Gupta, 'Torsional surface wave in an elastic half space with void pores', *Int. j. numer. anal. methods geomech.*, **17**, (1993).